Eur. Phys. J. D **35**, 541–545 (2005)
DOI: 10.1140/epjd/e2005-00067-y

THE EUROPEAN
PHYSICAL JOURNAL D

Virtual photon effects on classical chaos in a periodically kicked Rydberg atom system

X.L. Zhang[2], C.H. Yuan[1], H.B. Huang[1,a], and R. Ju[1]

[1] Department of Physics, Southeast University, Nanjing 210096, P.R. China
[2] Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, P.R. China

Received 10 March 2004 / Received in final form 5 May 2004
Published online 12 May 2005 –

Abstract. We study classical chaos in the system of a two-level Rydberg atom interacting with a pulsed standing microwave. This model approaches the form of an atom optics realization of a usual delta-kicked rotor under the rotating-wave approximation (RWA). We find that the non-energy-conserving processes or virtual photon processes neglected in the RWA have a strong effect on the classical chaos, which can enhance, reduce and even completely suppress the chaos under certain kicked conditions. The system displays non-KAM dynamical behavior for rational and irrational kicks.

PACS. 42.65.Sf Dynamics of nonlinear optical systems; optical instabilities, optical chaos and complexity, and optical spatio-temporal dynamics – 05.45.Ac Low-dimensional chaos

1 Introduction

In the study of classical chaos, a prototype model is the delta-kicked rotor (DKR) [1], whose classical dynamics is described by the well-known Chirikov standard map [2] (CSM). This is a 2D continuous perturbed twist map, with a transition point, discriminating between motion that is bounded (prevalently regular on invariant Kolmogorov-Arnold-Mose (KAM) tori) or unbounded and diffusive (prevalently chaotic). In the quantum case, the DKR can generate dynamical localization, namely, the quantum suppression of classical diffusion [3]. This and related models have been achieved in atom optics [4–7], microwave, and periodic train of impulses driven Rydberg atom systems [8–10]. But, to our knowledge, in all the atom optics studies of the DKR [4–7], the rotating-wave approximation (RWA) was used, and the counter-rotating terms, which correspond to highly non-energy-conserving processes or virtual photon processes were neglected. This is generally a very good approximation in quantum optics [11] and high atomic transition frequency regions. However, we should keep the counter-rotating terms or virtual photon processes in the strong interaction of micromaser and Rydberg atoms. The reason for this is that the transition frequency between the two Rydberg atomic levels concerned in the interaction is much lower than that in usual atom optics DKR where the atomic transition frequency is in the visible or near IR regions. Detailed discussion will be given in the following section.

[a] e-mail: hongbinh@seu.edu.cn

We should note that in the studies of nonlinear dynamics of microwave driven Rydberg atom systems so far, the "pendulum approximation" [3] was made, and the effect of electron transition between different energy levels on the time dependence of electron dipoles is neglected, so that the dipole matrix elements are proportional to the square of the principal quantum number of the electron orbit. This leads to the result that matrix elements are a slowly varying function of time. On the other hand, all these studies are in a center-of-mass system, and relative motion between electrons in Rydberg states and the atomic core are important. Since the diameters of Rydberg atoms are much less than the microwave wavelength, microwave pulses can be treated as a sinusoidal perturbation $\mathbf{E} = \mathbf{E}_0 \cos\omega_L t$. The momentum absorbed by Rydberg atoms from microwaves is therefore independent of the coordinates of the center-of-mass system. Furthermore, the pulse widths of the microwaves used in these studies were usually much larger than the period ν_L^{-1} of the microwave. From the above discussions we clearly see that the virtual photon processes have ignored effects on the dynamics in these studies.

In this paper we investigate the classical chaos in the system of a two-level Rydberg atom, which has large electron orbit and long lifetime, interacting with a pulsed standing microwave, in which the Rydberg atom is subjected to a train of standing microwave pulses with repetition frequencies $\nu_T = 1/T$ and pulse duration T_p. If the microwave frequency $\omega_L \gtrsim T_p^{-1} \gg \tau_n^{-1}(\tau_n$ is the Rydberg atom lifetime), the interaction between the microwave and Rydberg atom will be both time and position dependent, and the systems will exhibit richer classical and quantum dynamics than DKR. To the best of our knowledge, the dynamics of this model have not been studied thus far. The study of this model will provide new insight into classical dynamics, classical-quantum correspondence, and quantum dynamics.

2 The effect of virtual photon

The system we consider is the exact interaction of a cold two-level Rydberg atom with transition frequency ω_0 between its lower $|1\rangle$ and upper level $|2\rangle$, and a pulsed standing microwave $\mathbf{E} = \mathbf{E}_0 \cos(k_L x)\cos(\omega_L t)$. The exact Hamiltonian for the system is described by

$$H = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \frac{p^2}{2\mu} - \mathbf{D}\cdot\mathbf{E}_0(\sigma_+ + \sigma_-)\cos(\omega_L t)\cos(k_L x)\sum_{n=-\infty}^{\infty} f(t-nT), \quad (1)$$

where $\sigma_+ = |2\rangle\langle 1|$, $\sigma_- = |1\rangle\langle 2|$, and $\sigma_z = |2\rangle\langle 2| - |1\rangle\langle 1|$ are the Pauli operators, which satisfy the commutation relations:

$$[\sigma_+, \sigma_-] = \sigma_z, [\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm \quad (2)$$

and $f(t)$ specifies the temporal shape of the pulses. $\mathbf{p}$ is the center-of-mass momentum, and μ is the atomic mass. We transform the Hamiltonian into a frame rotating with the microwave frequency ω_L by a unitary transformation $T = e^{\frac{1}{2}i\omega_L t\sigma_z}$

$$\begin{aligned} H &= THT^+ - i\hbar T\dot{T}^+ \\ &= \frac{p^2}{2\mu} + \frac{1}{2}\hbar\Delta\sigma_z \\ &\quad -\hbar\Omega\cos(k_L x)\cos(\omega t)(\sigma_+ e^{i\omega_L t} + \sigma_- e^{-i\omega_L t}) \\ &= \frac{p^2}{2\mu} + \frac{1}{2}\hbar\Delta\sigma_z - \frac{1}{2}\hbar\Omega\cos(k_L x)(\sigma_+ + \sigma_-) \\ &\quad -\frac{1}{2}\hbar\Omega\cos(k_L x)(\sigma_+ e^{i2\omega_L t} + \sigma_- e^{-i2\omega_L t}) \end{aligned} \quad (3)$$

where $\Omega = \mathbf{D}\cdot\mathbf{E}_0/\hbar$ is the resonant Rabi frequency, and $\Delta = \omega_0 - \omega_L$ is the detuning. For convenience, the term $\sum_n f(t-nT)$ is dropped for the time being. In the RWA, the counter-rotating term, that is the last term containing $e^{\pm i2\omega_L t}$ is neglected if the coupling is weak [11]. It is unimportant because in quantum optics, the time scales are usually large compared with ν_L^{-1}, so this term will be approximately averaged to zero. But, in the present case, the width of the microwave pulses T_p can reach the period ν_L^{-1} of the microwave. So the counter-rotating terms must be kept. The Hamiltonian H can be diagonalized in atomic state space ($|1\rangle$, $|2\rangle$) and written as:

$$\bar{H} = \frac{p^2}{2\mu} + \sqrt{\frac{1}{4}\hbar^2\Delta^2 + \hbar^2\Omega^2\cos^2(k_L x)\cos^2(\omega_L t)}\ \sigma_z \quad (4)$$

In the limit of large detuning $\Delta^2 \gg \Omega^2$, $\bar{H}$ can be written as:

$$\bar{H} = \frac{p^2}{2\mu} + \left[\frac{1}{2}\hbar\Delta + 2K + 2K\cos(2\omega_L t) + 4K\cos^2(\omega_L t)\cos(2k_L x)\right]\sigma_z. \quad (5)$$

Here $K = \hbar\Omega^2/8\Delta$ is the coupling constant. The terms $\frac{1}{2}\hbar\Delta + 2K + 2K\cos(2\omega_L t)$ can be removed by a unitary transformation $U = \exp\{[i(\frac{1}{2}\Delta + \frac{2K}{\hbar})t + \frac{iK}{\hbar\omega_L}\sin(2\omega_L t)]\sigma_z\}$

$$\begin{aligned} \bar{H}' &= U\bar{H}U^+ - i\hbar U\dot{U}^+ \\ &= \frac{p^2}{2\mu} + 4K\cos^2(\omega_L t)\cos(2k_L x)\sigma_z. \end{aligned} \quad (6)$$

From the Hamiltonian we can obtain two uncoupled differential equations for the excited and ground state amplitudes. Assuming the Rydberg atom is initially in its ground state, then we may neglect the excited state amplitude, and obtain the following Hamiltonian for the impulsively driven (kicked) Rydberg atom

$$H = \frac{J^2}{2I} + 4K\cos^2(\omega_L t)\cos\theta\sum_{n=-\infty}^{\infty} f(t-nT). \quad (7)$$

Here $J = p/2k_L$, $\theta = 2k_L x$ and $I = \mu/4k_L^2$. Equation (7) reduces to the result of the RWA [6] if the term $4\cos^2(\omega_L t)$, which is caused by virtual photon processes, is dropped. All new phenomena discussed in this paper come from this time dependent coupling coefficient, that is the virtual photon processes. It is obvious that the optical potential is time modulated by the virtual photon processes. Before discussing the classical properties of this Hamiltonian, several points should be stressed. Since the recoil frequency $\omega_r = \hbar k_L^2/2\mu = 1.25\times 10^{-2}/A\lambda_L^2$ (A is the Rydberg atom mass in a.u., and λ_L is the wavelength of the microwave in cm unit) of the Rydberg atom is much less than $2\pi/T_p$, that is $\omega_r T_p \ll 2\pi$, the Rydberg atom is in the Raman-Nath regime, and the motion of the Rydberg atom during the interaction is negligible. Thus the interaction of a Rydberg atom with a potential is equivalent to the elastic collision between an atom and a photon. After the interaction, the atom will receive a momentum increase $\triangle p \simeq 8Kk_L T_p\cos^2(\omega_L T_p)\sin\theta$. On the other hand, we can choose $\omega_L \sim 2\pi/\mathcal{T}_n = n^{-3}$ (where $\mathcal{T}_n = 2\pi n^3$ (a.u.) is the electron classical orbit period, in an energy level with principal quantum number n), so in the classical picture, the electron in a large orbit can respond to the microwave and absorb the momentum $\Delta\mathbf{p} = -\int\mathbf{E}(t)dt$ (a.u.) from the pulses, that is the electron is "kicked" periodically. We should note that the periodical "kicks" of the electron by the microwave may cause the internal chaotic motion of the electron in a Coulomb orbit [10]. But this internal chaotic motion is negligible for $\triangle^2 \gg \Omega^2$ and a small Rydberg atomic orbit n (τ_n is small compared with the characteristic time of the electron motion in orbit n).

$\Delta\mathbf{p}$ can be calculated for a Gaussian pulse

$$f(t-nT)=\frac{1}{T_p\sqrt{2\pi}}e^{-\frac{(t-nT)^2}{2T_p^2}}$$

(the specific temporal shape $f(t)$ of the pulse has no effect on the following conclusion), and $T_p > \nu_L^{-1}$, which gives $\Delta\mathbf{p} = -\int_0^{T_p}\mathbf{E}(t)dt \approx \int_{-\infty}^{\infty}\mathbf{E}(t)dt = e^{-\frac{1}{2}T_p^2\omega_L^2}\mathbf{E}_0\cos(n\omega_L T)\cos(k_L x)$. It is clear that $\Delta\mathbf{p}=0$ for $\omega_L T_p \gg 1$, in this case the RWA should be used. However, we should not make the RWA for $\omega_L T_p \gtrsim 1$, and the counter-rotating term should be kept. This is just the case in an impulsively driven Rydberg atom system. In the limit of arbitrarily short pulses, this system is equivalent to the DKR with a time dependent coupling constant caused by virtual photon processes. The nonzero pulse widths lead to a reduction of the coupling strength with increasing T_p. For simplicity, we let $f(t-nT)=\delta(t-nT)$ in the following discussion. From equation (7) we have the following generalized standard map (GSM):

$$J_{n+1}=J_n+4K\cos^2(n\omega_L T_n)\sin(\theta_n)$$
$$\theta_{n+1}=\theta_n+J_{n+1}T_{n+1}/I. \qquad (8)$$

The terms $4K\cos^2(n\omega_L T_n)$ completely specify the effect of the virtual photons on the classical dynamics. It is obvious that the map is not an autonomous map, so the orbits can intersect. For $\omega_L T_n = m\pi$ (m = integer), $T_n = T_{n+1}$ and $4K_c \geq 1$, the classical dynamics are globally chaotic. In this case we see that the non-energy-conserving processes lead to chaos even for weak coupling $K_c \geq 1/4$. If $\omega_L T_n = (m \pm \beta/n)\pi$, the virtual photon processes can enhance, compared with the results of RWA, the classical chaos for $0 \leq \beta < 1/3$, and can suppress the chaos for $1/3 < \beta < 2/3$. Notably, the dynamics of the system under RWA are the same as that of the system without RWA for $\beta = 1/2$. The point to be stressed is that the virtual photon processes can even completely suppress the classical chaos for $\omega_L T_n = \pi(m \pm 1/2n)$, a phenomenon that can not occur in the RWA.

In order to numerically study the effect of virtual photons on the classical dynamics, we transform equation (8) into scaled dimensionless form $\rho_{n+1} = \rho_n + 4\overline{K}\cos^2(n\overline{\omega}_L)\sin\theta_n$; $\theta_{n+1} = \theta_n + \rho_{n+1}$ with $\tau = t/T$, $\rho = JT/I$, $\overline{\omega}_L = \omega_L T$, and $4\overline{K} = 4K(8\omega_r T^2/\hbar) = 0.1T^2\Omega^2/(2\Delta A\lambda_L^2)$. In the following discussions we focus on two cases: $\overline{\omega}_L = (m \pm \alpha)\pi$ $(0 < \alpha < 1)$, where α is an irrational number, and $\overline{\omega}_L = (m \pm p/q)\pi$ (m = integer, $0 < p/q \leq 1$), where p/q is a rational fraction. For very small $\overline{K}$, in the CSM most of the map is composed of KAM tori.

But in the GSM, if $\overline{\omega}_L = (m \pm \alpha)\pi$, the invariant curves do not exist at all, even for very small $\overline{K}$ (see Fig. 1). The system is turned into another class of systems which is non-KAM [12,13]. In Figure 2, we demonstrate a few phase maps at different values of $\overline{K}$. It is clearly seen that the system is non-KAM and non-islet for any $\overline{K}$. Different stochastic webs are the main characteristic of this system. Thus diffusion can take place

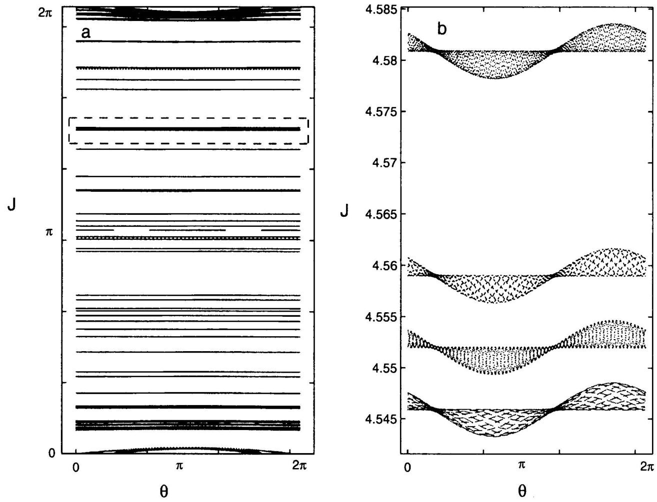

Fig. 1. Classical phase space of GSM for $\overline{K} = 0.001$, $\overline{\omega}_L = (m \pm 1/7\pi)\pi$. Figure (b) shows a magnification of the dashed square in (a).

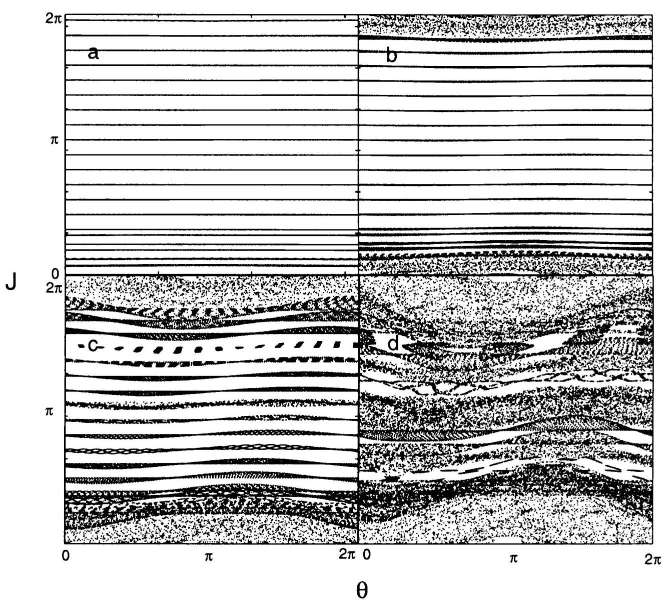

Fig. 2. Classical phase space of GSM for $\overline{\omega}_L = (m \pm 1/7\pi)\pi$ and several values of $\overline{K}$: (a) $\overline{K} = 0.001$; (b) $\overline{K} = 0.01$; (c) $\overline{K} = 0.05$; (d) $\overline{K} = 0.15$.

along the stochastic webs for any small $\overline{K}$. As $\overline{K}$ increases, the band structured stochastic layer becomes wider and wider, and eventually covers the whole phase space. This comes from the fact that $\cos^2(n\alpha\pi)$ is not a periodic function of n, and so do ρ and θ. If $\overline{\omega}_L = (m \pm p/q)\pi$, most of the map is composed of KAM tori. Since $\cos^2(np\pi/q) = \cos^2(p\pi/q), \cos^2(2p\pi/q), ..., \cos^2(p\pi)$ has q values in one map period, so one KAM curve in CSM now becomes q KAM curves and these KAM curves entangle (see Fig. 3b). The cross points generated by the entangled KAM curves are not hyperbolic fixed points, but the chaos firstly takes place in the neighborhood of the cross points as $\overline{K}$ increases. Only period q fixed points

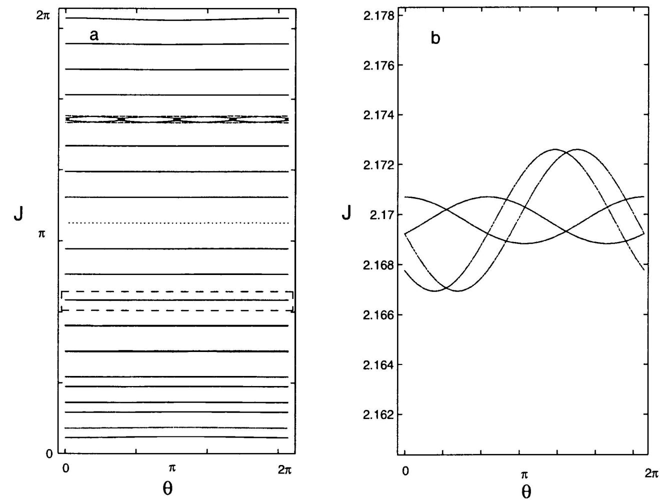

Fig. 3. Classical phase space of GSM for $\overline{K} = 0.001$, $\varpi_L = (m \pm 1/4)\pi$. Figure (b) shows a magnification of KAM curves in the dashed square in (a).

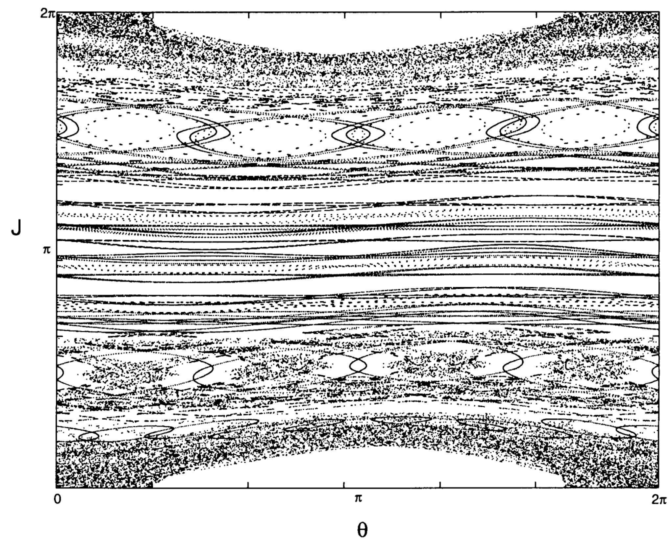

Fig. 4. Classical phase space of GSM for $\overline{K} = 0.05$, $\varpi_L = (m \pm 1/4)\pi$.

are founded in GSM, the positions of the period q fixed points are at $\rho = (1/q)2\pi$ and $(1 - 1/q)2\pi$ (see Fig. 4), unlike in CSM, the elliptic periodic orbits of different fixed points, in most situations, cross each other (see Fig. 4). When $\overline{K}$ increases, the elliptic periodic orbits separate and then form high order elliptic periodic orbits as $\overline{K}$ increases further. Eventually chaotic orbits cover the whole phase space. The point to be stressed is that other chaotic orbits can enter into the elliptic orbits, which never occurs in CSM as shown in Figure 4. We should note that the system approaches non-KAM system as $q \longrightarrow \infty$. The reason for this is that the period of $\cos^2(np\pi/q)$ increases with increasing q, and eventually $\cos^2(np\pi/q)$ becomes an aperiodic function of n.

At this point, it is natural to ask to what extent the Rydberg atoms behave as classical particles? We now briefly discuss this question. From the quantized model, θ and $\rho = JT/I$ satisfy the commutation relation $[\theta, \rho] = i\kappa$, where the scaled Planck constant $\kappa = 8\omega_r T \backsimeq 0.1T/A\lambda_L^2 \backsimeq 8.3 \times 10^{-5}$ for $T = 80$ μs, $\lambda_L = 0.31$ cm, and $A = 1$. This very small value of the scaled Planck constant certainly suggests that the system behaves classically. In order to reveal the conditions under which the system behaves quantum mechanically, we should calculate the dynamical localization. Detailed discussions on this subject will be presented elsewhere.

The results discussed above could be achieved in the following proposed experiment [14]. We first trap and laser cool neutral atoms (for example hydrogen) in a magneto-optic trap (MOT), and then photo-excite the atoms to create a Rydberg atom with large effective electron orbit $n^* \approx 40$. The MOT is then turned off, a pulsed standing microwave with $\nu_L = 97$ GHz, $\Delta = 6$ GHz, $T = 80$ μs, $\Omega = 800$ MHz, and $T_p \gtrsim \nu_L^{-1}$ (from these parameters we obtain $\overline{K} \simeq 0.1$, which is large enough for our present studies) is turned on, during which the microwave delivers an impulsive momentum transfer (or "kick") to the Rydberg atoms and causes diffusive growth in the center-of mass momentum. After the standing microwave is turned off, the Rydberg atoms undergo free expansion in the dark for a time $t_0 < \tau_n$, and then the atoms pass through a resonant probe laser beam, which has been used to create the Rydberg atom state n^*, and the amount of absorption in the beam is measured by cooled CCD. We can also use a resonant microwave to probe the atoms, the amount of absorption $n^* \rightarrow n^* + 1$ in the beam can be measured by an array of high frequency diodes. The free evolution time t_0 is measured by the time-of-flight technique. From the initial MOT size, the time t_0, and the spatial absorption spectrum we can determine the atomic momentum distribution. The noise in the wings of the momentum distribution can be reduced by filtering the time-of-flight measurements using a Fourier technique.

3 Conclusion

In conclusion, we have studied the classical behavior of a Rydberg atom interacting with a pulsed standing wave of light, which is a generalization of the RWA result. The virtual photon processes neglected in the RWA, can enhance, reduce and even completely suppress classical chaos under certain kicked conditions. The system is a non-KAM system if $\cos^2(n\omega_L T)$ is not a period function of n, and is a KAM system if $\cos^2(n\omega_L T)$ is a period function of n.

We would like to thank the referee for their valuable comments.

References

1. G. Casati, B.V. Chirikov, F.M. Izraelev, J. Ford, *Lecture Notes in Physics* (Springer-Verlag, Berlin, 1979), Vol. 93, p. 334
2. B.V. Chirikov, Phys. Rep. **52**, 263 (1979); D.L. Shepelyansky, Physica D **8**, 208 (1983)

3. L.E. Reichl, *The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations* (Springer-Verlag, Berlin, 1992)
4. R. Graham, M. Schlautmann, P. Zoller, Phys. Rev. A **45**, R19 (1992)
5. F.L. Moore, J.C. Robinson, C.F. Bharucha, Sundaram Bala, M.G. Raizen, Phys. Rev. Lett. **75**, 4598 (1995)
6. B.G. Klappauf, W.H. Oskay, D.A. Steck, M.G. Raizen, Phys. Rev. Lett. **81**, 4044 (1998)
7. H. Ammann, R. Gray, I. Shvarchuck, N. Christensen, Phys. Rev. Lett. **80**, 4111 (1998); M.B. d'Arcy, R.M. Godun, M.K. Oberthaler, D. Cassettari, G.S. Summy, Phys. Rev. Lett. **87**, 074102 (2001)
8. G. Casati et al., Phys. Rep. **154**, 77 (1987); G. Casati, I. Guarneri, B.A. Shepelyansky, IEEE J. Quant. Electron **24**, 1420 (1988); J.E. Bayfield, D. Casati, I. Guarneri, D. Sokol, Phys. Rev. Lett. **63**, 364 (1989); R.V. Jensen et al., Phys. Rep. **201**, 1 (1991)
9. E.J. Galvez, B.E. Saver, L. Moorman, P.M. Koch, D. Richard, Phys. Rev. Lett. **61**, 2011 (1988); C.F. Hillermeier, R. Blümel, U. Smilamsky, Phys. Rev. A **45**, 3486 (1992); P.M. Koch, *in Chaos and Quantum Chaos, Proceedings of the Eighth South African School in Physics*, (Springer-Verlag, Berlin, 1993); P.M. Koch et al., Phys. Rep. **255**, 290 (1995)
10. C.O. Reinhold, J. Burgdörfer, M.T. Frey, F.B. Dunning, Phys. Rev. Lett. **79**, 5226 (1997); S. Yoshicla, C.O. Reinhold, J. Burgdörfer, Phys. Rev. Lett. **84**, 2602 (2000)
11. W.H. Louisell, *Quantum Statistical Properties of Radiation* (John Wiley & Sous, New York, 1973); M.O. Scully, M.S. Zubairy, *Quantum Optics* (Cambridge, 1997)
12. Borgonovi Fausto, Phys. Rev. Lett. **80**, 4653 (1998)
13. Hu Bambi, Li Baowen, Liu Jie, Gu Yan, Phys. Rev. Lett. **82**, 4224 (1999)
14. R.M. Goduu, M.B. d'Arcy, M.K. Oberthaler, G.S. Summy, K. Burnett, Phys. Rev. A **62**, 013411 (2000); M.B. d'Arcy et al., Phys. Rev. Lett. **87**, 074102-1 (2001)